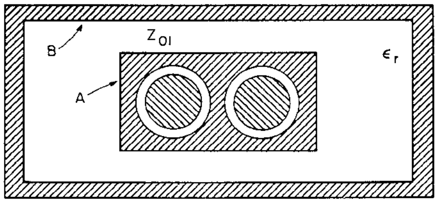

Fig. 1 Re-entrant coaxial directional coupler (Z_{01} is the characteristic impedance of the coaxial line with center conductor A and outer conductor B).

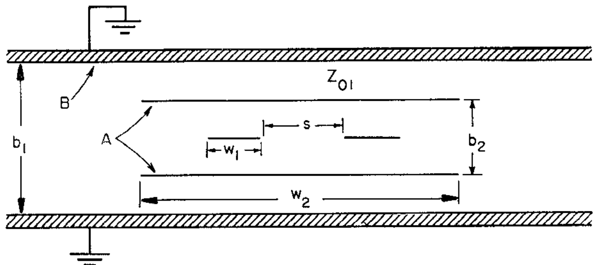

Fig. 2. Strip transmission line re-entrant directional coupler. (Z_{01} is the characteristic impedance of the coaxial line with center conductor A and outer conductor B) (Constraint: $w_2 \geq 2w_1+s+2b_2$.)

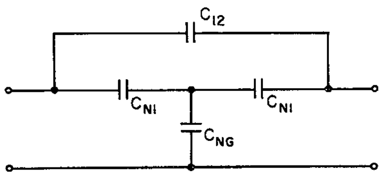

Fig. 3. Capacitance network at an arbitrary cross section of a re-entrant coupler with direct coupling between center conductors.

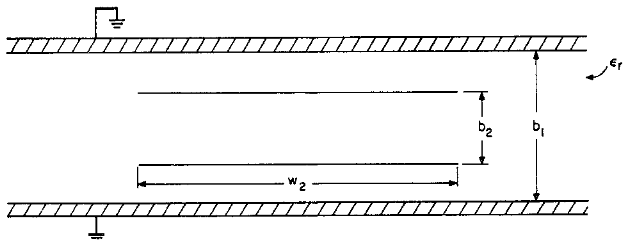

Fig. 4. Network for determination of C_{NG} and Z_{01}.

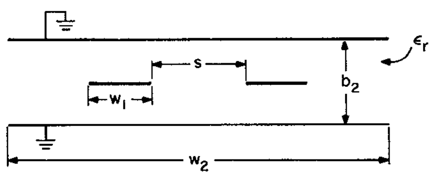

Fig. 5. Network for determination of C_{N1} and C_{12}.

Equation (15) corresponds to

$$\sqrt{\epsilon_r} Z_{01} = 79.7. \qquad (16)$$

This is a 5.31-percent reduction in the required Z_{01}. In a similar calculation, a value of $C_{12}/\epsilon = 3$ gave a 15.3-percent reduction in the required Z_{01}.

Conclusion

Directional couplers of re-entrant cross-sectional and direct coupled center conductors give smaller required values of Z_{01} and, therefore, may often permit easier realization of Z_{01} than similar couplers without direct coupled center conductors.

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[3] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission-line filters and directional couplers," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-4, pp. 75–81, April 1956.

On Incidental Dissipation in High-Pass and Band-Stop Filters

For narrow-band microwave band-stop filters, Young, et al.[1] have obtained valuable expressions for the effects of incidental dissipation. The purpose of this correspondence is to remark that for the uniformly dissipative (i.e., equal Q) case, the response of the lossy low-pass prototype (and hence, the response of the lossy band-stop filter) may be obtained from the lossless response by means of the frequency transformation.

$$p = \frac{s}{1 + ds} \qquad (1)$$

where

$$d = \frac{1}{wQ}$$

and p and s are the complex frequency variables for the lossless and lossy cases, respectively. Also, w is the normalized pass bandwidth defined by Young, et al.,[1] and Q is the quality factor of each resonator of the band-stop filter.

To obtain (1), it is merely necessary to observe that each lossy resonator of the band-stop filter (Fig. 8 of Young et al.[1]) corresponds, in the low-pass prototype, to either an inductor and a resistor in parallel or a capacitor and a resistor in series, and then to carry out the details of the frequency transformation.

For estimation of pass band losses, (1) may be put in more convenient form. From (1), obtain

$$= \frac{s - ds^2}{1 - (ds)^2}$$

and, for $s = j\omega$,

$$p = \frac{d\omega^2 + j\omega}{1 + (d\omega)^2}. \qquad (2)$$

For $d\omega \ll 1$

$$p \approx d\omega^2 + j\omega. \qquad (3)$$

Thus, at frequencies for which (3) is valid, the lossy low-pass prototype may be considered to have a frequency-dependent dissipation factor $\delta = d\omega^2$. It follows that the loss may be computed to first order from the group delay according to the well-known relation[2], which becomes

$$L_d(\omega) = A_0(\omega) + 8.686 d\omega^2 T_g(\omega). \qquad (4)$$

Here, $A_0(\omega)$ and $T_g(\omega)$ are, respectively, the attenuation and group delay of the lossless low-pass prototype, while L_d is the attenuation (decibels) of the lossy low-pass prototype.

Equation (4) is not exact, due to both approximation (3) and the error involved in truncating the Taylor series expression for L_d, but for small d, these errors should not be large. For the equal Q case only, then, (4)

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[1] L. Young, G. L. Matthaei, and E. M. T. Jones, "Microwave band-stop filters with narrow stop bands," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-10, pp. 416–427, November 1962.

[2] H. W. Bode, *Network Analysis and Feedback Amplifier Design*. Princeton, N. J.: Van Nostrand, 1945, pp. 216–222.

may be considered as an alternate to the approximation given in Fig. 8 of Young et al.[1]

Finally, it may be noted that by redefining d, (1) through (4) may be applied to the high-pass case. Moreover, this approach is not limited to microwave filter applications, and it seems likely that it has been used by others. However, the author has not seen it in print.

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Multisection Microwave Phase-Shift Network

This correspondence extends the analysis of a phase-shift network consisting of a cascade of pairs of coupled transmissions lines connected together at their far ends[1] (see Fig. 1), to any value of n. Such cascaded all-pass networks, also known as microwave C-sections, have recently been analyzed by Steenart [1] and Zysman and Matsumoto [2]. Cristal [3] has solved the problem of analysis and exact synthesis of cascaded C-sections. Matrix methods are generally used and Richards' theorem [4] is employed, thereby restricting the various coupled sections to have equal lengths. In this analysis, the various sections may have unequal lengths.

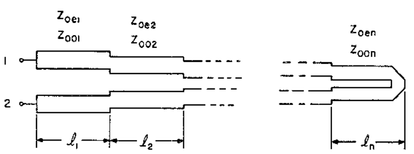

Fig. 1.

Earlier, Jones and Bolljahn [5] gave a formula for the phase shift ϕ through a single C-section terminating in $Z_0 = (Z_{0o}Z_{0e})^{1/2}$, where Z_{0o} and Z_{0e} are the odd and even mode impedances, respectively, of the coupled-lines. Thus, for the single microwave C-section

$$\phi = \cos^{-1}\left(\frac{\rho - [\tan\theta]^2}{\rho + [\tan\theta]^2}\right) \qquad (1)$$

where $\rho = Z_{0e}/Z_{0o}$, and $\theta = 2\pi l/\lambda$ is the electrical length of the coupled section of physical length l, and λ is the wavelength in the medium. Dr. S. B. Cohn first suggested the use of the coupled-line all-pass network in broadband 90-degree differential phase-shift networks, which lead to a work of the writer containing the formula for the two-section ($n=2$) phase-shift network [6],

$$\phi(n=2) = \cos^{-1}\left(\frac{\rho_1 - [\tan\theta_1']^2}{\rho_1 + [\tan\theta_1']^2}\right) \qquad (2)$$

where

$$\theta_1' = \theta_1 + \tan^{-1}\left[(Z_{0o2}/Z_{0o1})\tan\theta_2\right].$$

Here the subscripts 1 and 2 refer to the first and second coupled sections. At the same time, the writer derived, but did not publish, the formula for the three-section coupled-line phase-shift network, which is given below

$$\phi(n=3) = \cos^{-1}\left(\frac{\rho_1 - [\tan\theta_1']^2}{\rho_1 + [\tan\theta_1']^2}\right) \qquad (3)$$

where

$$\theta_1' = \theta_1 + \tan^{-1}\left[(Z_{0o2}/Z_{0o1})\tan\theta_2'\right],$$

and

$$\theta_2' = \theta_2 + \tan^{-1}\left[(Z_{0o3}/Z_{0o2})\tan\theta_3\right].$$

Equations (1) to (3) now suggest the following formula for the general n-section cascade of microwave C-sections:

$$\phi_n = \cos^{-1}\left(\frac{\rho_1 - [\tan\theta_1']^2}{\rho_1 + [\tan\theta_1']^2}\right) \qquad (4)$$

where

$$\begin{aligned}\theta_1' &= \theta_1 + \tan^{-1}(\sigma_{12}\tan\theta_2'),\\ \theta_2' &= \theta_2 + \tan^{-1}(\sigma_{23}\tan\theta_3'),\\ &\vdots\\ \theta_{n-1}' &= \theta_{n-1} + \tan^{-1}(\sigma_{(n-1),n}\tan\theta_n'),\\ \theta_n' &= \theta_n,\end{aligned}$$

and

$$\sigma_{i,i+1} = (Z_{0ei}/Z_{0e(i+1)}) = (Z_{0o(i+1)}/Z_{0oi}).$$

The foregoing result holds only when $(Z_{0oi}Z_{0ei}) = Z_0^2$.

Equations (2) and (3) were obtained as follows. An even-mode input of $(+\frac{1}{2}, +\frac{1}{2})$ volt from a pair of Z_0 sources was assumed at Ports 1 and 2, with the far end open-circuited. Likewise, an odd-mode input of $(+\frac{1}{2}, -\frac{1}{2})$ volt was assumed at Ports 1 and 2, respectively, with a short-circuit termination at the far end. The two normal-mode inputs add to a coupled-mode input of $(+1, 0)$ volt at Ports 1 and 2, which means that power from the generators is flowing into Port 1 but not Port 2. We now require that the output wave be $(0, e^{j\phi})$, which is the condition for a perfect match, and which also is in conformity with the principle of the conservation of energy. This condition on the output wave may also be put in terms of the even-mode and odd-mode reflection coefficients

$$(\Gamma_e + \Gamma_o) = 0 \qquad 1/2(\Gamma_e - \Gamma_o) = e^{j\phi}. \qquad (5)$$

The reflection coefficients Γ_e and Γ_o were then found with the aid of transmission-line theory, and it was readily ascertained that the desired conditions (5) could be met by letting $(Z_{0ei}Z_{0oi}) = Z_0^2$ for all i. The two expressions (5) in Γ were then combined and solved for the phase of the output wave at Port 2,

$$\phi = \cos^{-1}[\mathrm{Re}\,(\Gamma_e)] \qquad (6)$$

yielding (2) and (3).

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[1] The original work [6] was sponsored by the Air Force Cambridge Research Center, Bedford, Mass., under Contract AF 19(604)-1571.

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Dual Reflex Klystron Cavity-Beam Interaction

While the occurrence of spurious resonances, or the detection of harmonic content, is not uncommon in reflex oscillators, the particular case wherein the tube performs reflex klystron functions at two different frequencies has not been reported heretofore. This phenomenon was observed in the course of a search for spurious output signals conducted on Raytheon QKK754 CW communication klystrons. X-band output was detected from these tubes using the RF equipment shown in Fig. 1.

The amplitude of the X-band signal was strongly dependent on external loading. Under optimum C-band load conditions, its power level was 25 to 27 dB lower than the C-band output of the tubes. The spurious power content rose by 6 to 8 dB when the load was adjusted for maximum output at X-band.

Upon sweeping the reflector with 60-cycle ac voltage, it was possible to monitor the $2\frac{3}{4}$ and $3\frac{3}{4}$ C-band and the $5\frac{3}{4}$, $6\frac{3}{4}$, and $7\frac{3}{4}$ X-band reflector modes simultaneously. These modes are presented on the same trace and separately in Fig. 2, to simplify viewing. The dual reflex klystron processes are evident from these oscilloscope traces.

As the plots of Fig. 3 reveal, gap tuning of the C-band resonator resulted in continuous tuning of the X-band mode over most of the mechanical tuning range of the tube. The nonintersecting reflector voltage curves provide a clear indication that there was no simultaneous X-band oscillation in the tubes at the peak of the $2\frac{3}{4}$ C-band reflector mode.

Further study of the dual cavity-beam interaction suggested that the reentrant cavity contained an interacting resonance in the first cutoff or TM_{011} coaxial mode, shown in Fig. 4, in addition to the regular TM_{010} mode.

To confirm this, the resonant frequency of the TM_{011} mode was computed along the lines developed by MacKenzie.[1] The result-

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[1] L. A. MacKenzie, "Klystron cavities for minimum spurious output power," Cornell Research Rept. EE-418, School of Electrical Engineering, Cornell Univ., Ithaca, N. Y., January 1959.